(a) $\lim\limits_{x \to -3} \dfrac{x^2 - 3x - 18}{x + 3}$

(b) $\lim\limits_{x \to 0} \dfrac{x + \sin x}{x}$

(c) $\lim\limits_{x \to 0} \dfrac{3 \sin 4x}{\sin 3x} \cdot \dfrac{4x}{4x} = 4 \cdot \dfrac{3x}{\sin 3x} \cdot \dfrac{\sin 4x}{4x} = 4 \cdot \dfrac{\sin 4x}{4x} \div \dfrac{\sin 3x}{3x}$

$4 \cdot 1 \div 1 = 4$

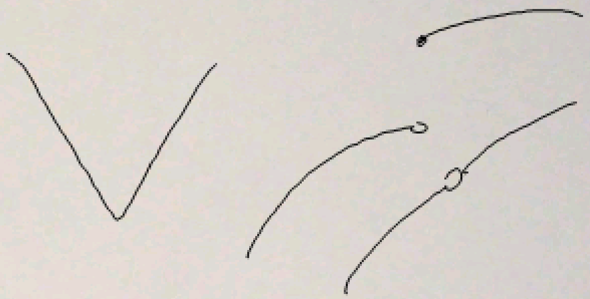
(d) $\lim\limits_{x \to 3} \dfrac{2x^3 - 4x^2 - 18}{x - 3}$

$$\begin{array}{r|rrrr} 3 & 2 & -4 & 0 & -18 \\ & & 6 & 6 & 18 \\ \hline & \multicolumn{3}{l}{2x^2 + 2x + 6} & 0 \end{array}$$

Course Review

Chapter 2:

- Average vs. Instantaneous speed, p. 59
- Limits
    - Properties p. 61 & 71
    - One-sided and Two-sided limits – the limit at a point exists only if the right-hand and left-hand limits exist and are equal, p. 63
- Continuity, p. 79
- Intermediate Value Theorem, p. 83
- Tangent and Normal Lines, pp. 88-91



Chapter 3:

- Derivative as the limit of the difference quotient, p. 99
- One-sided Derivatives, p. 104
- Differentiability, p. 109
    - Implies Continuity, p. 113
    - Intermediate Value Theorem for Derivatives, p. 113
- Derivative Rules pp. 116-121, pp. 141-145, p. 149, pp. 165-169, pp. 173-176

  [you have to know these, no ifs, ands or buts]
- order derivatives, p. 122

rules for derivitives from sect. 3.3.

$$f(x) = \frac{2x^2+1}{x+2}$$

$y = m(x - x_1) + y_1$

Slope $= f'(0) = -3$

pt. $\Rightarrow (0, f(0))$

$(0, -1)$

(4) Find the equations for both the tangent line and normal line to $f(x) = x^2 - 3x - 1$ at $x = 0$.

$2x - 3 \longrightarrow -3$

$y = -3x - 1$

$y = \frac{1}{3}x - 1$

(5) What is the rate of change of the volume of a cone with respect to the radius when the radius is 4 inches. The height of the cone is equal to the diameter and $V_{cone} = \frac{1}{3}\pi r^2 h$.

- Intermediate Value Theorem, p. 83
- Tangent and Normal Lines, pp. 88-91

Chapter 3:

- Derivative as the limit of the difference quotient, p. 99
- One-sided Derivatives, p. 104
- Differentiability, p. 109
    - Implies Continuity, p. 113
    - Intermediate Value Theorem for Derivatives, p. 113
- Derivative Rules pp. 116-121, pp. 141-145, p. 149, pp. 165-169, pp. 173-176

    [you have to know these, no ifs, ands or buts]
- Higher order derivatives, p. 122
- Instantaneous Rates of Change, p. 127
- Implicit Differentiation, p. 157

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

y'' y'''

Chapter 4:

- Extrema, pp. 187-189
- Critical Points, p. 190
- Mean Value Theorem, p. 196
- Increasing/Decreasing functions, p. 198
- Antiderivative, p. 200
- erivative Test, p. 205

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3) Find $\frac{dy}{dx}$ for each equation below.

a) $xy = \sin(x) + y^2$

$$x\frac{dy}{dx} + y\cancel{\frac{dx}{dx}} = \cos(x)\cancel{\frac{dx}{dx}} + 2y\frac{dy}{dx}$$

b) $y = \sin(x^3 - 5x + 1)$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = \cos(x) - y$$

c) $y = (x^3 - 1)\cos(x)$

$$\frac{dy}{dx} = \frac{\cos(x) - y}{x - 2y}$$

d) $y = \frac{2x+5}{3x-1}$

- Increasing/Decreasing functions, p. 198
- Antiderivative, p. 200
- Derivative Test, p. 205

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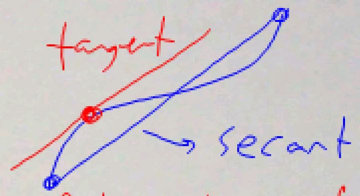
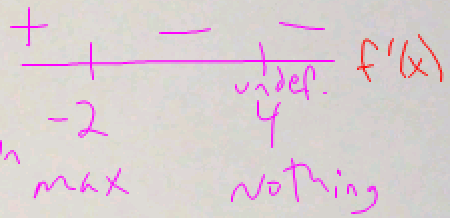
- Instantaneous Rates of Change, p. 127
- Implicit Differentiation, p. 157

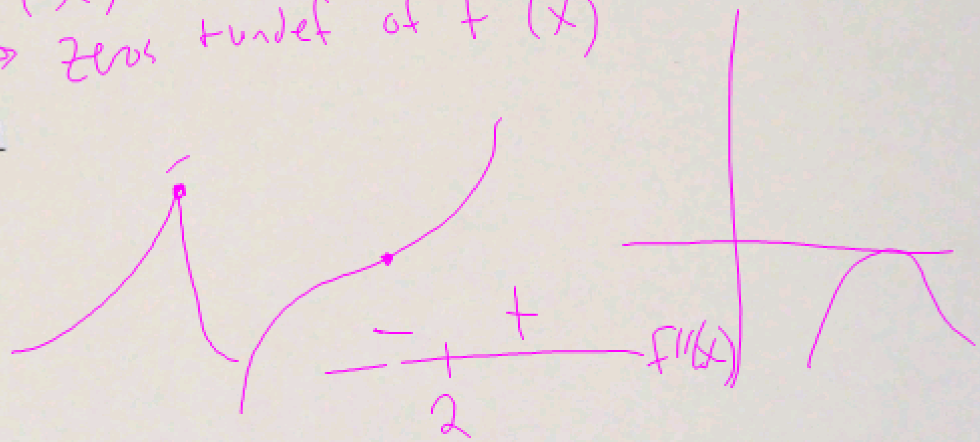


m of tangent = m of sec.

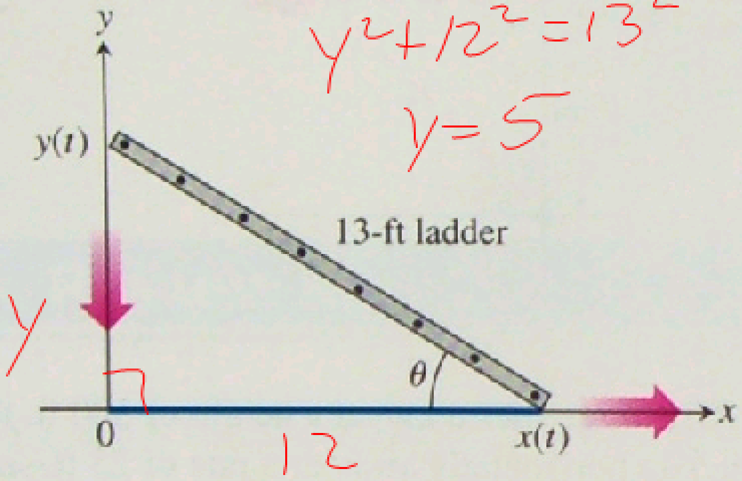
Chapter 4:

- Extrema, pp. 187-189 → [ ] absolute vs. local relative
- Critical Points, p. 190 → endpoints or zeros + undef. of $f'(x)$ → possible min/max
- Mean Value Theorem, p. 196
- Increasing/Decreasing functions, p. 198 ($f'(x)>0$ increasing, $f'(x)<0$ decreasing)
- Antiderivative, p. 200
- First Derivative Test, p. 205
- Concavity, p. 207 → $f''(x)>0$ up, $f''(x)<0$ down
- Inflection Points, p. 208 → zeros + undef of $f''(x)$
- Second Derivative Test, p. 211
- Optimization, p. 219
- Linearization, p. 233
- Differentials, p. 237
- Related Rates, p. 246





19. ***Sliding Ladder*** A 13-ft ladder is leaning against a house (see figure) when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at the rate of 5 ft/sec.



$y^2 + 12^2 = 13^2$

$y = 5$

$x^2 + y^2 = c^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2c\frac{dc}{dt}$

$2 \cdot 12 \cdot (5) + 2(y)\frac{dy}{dt} = 0$

$120 + 10\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-120}{10}$

$\frac{dy}{dt} = -12$ ft/sec

rate $= -12$ ft/sec

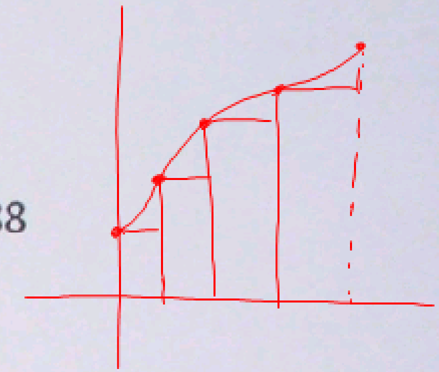
(a) How fast is the top of the ladder sliding down the wall at that moment? + 12 ft/sec

(b) At what rate is the area of the triangle formed by the ladder, wall, and ground changing at that moment? $-\frac{119}{2}$ ft^2/sec

(c) At what rate is the angle θ between the ladder and the ground changing at that moment? -1 radian/sec

Chapter 5:

- Rectangular Approximation Method (LRAM, RRAM, MRAM), p. 265
- Riemann Sums, p. 274
- Definite Integral, p. 276
- Area under a curve (as a definite integral), p. 278
- Rules for Definite Integrals, p. 285
- The Mean Value Theorem for Definite Integrals, p. 288
- Fundamental Theorem of Calculus p. 294 & 299
- Finding Total Area, p. 301
- Trapezoidal Rule, p. 307
- Simpson's Rule, p. 309



Chapter 6:

- Solving Differential Equations, p. 321, p. 350 (separation of variables)
- Slope Fields, p. 323
- Indefinite Integral
    - Properties of Indefinite Integrals, p. 332
    - U-substitution, p. 333
- Exponential Growth, p. 351
- Half-life, p. 353
- Newton's Law of Cooling, p. 355

Chapter 5:

- Rectangular Approximation Method (LRAM, RRAM, MRAM), p. 265
- Riemann Sums, p. 274
- Definite Integral, p. 276
- Area under a curve (as a definite integral), p. 278
- Rules for Definite Integrals, p. 285
- The Mean Value Theorem for Definite Integrals, p. 288
- Fundamental Theorem of Calculus p. 294 & 299
- Finding Total Area, p. 301
- Trapezoidal Rule, p. 307
- Simpson's Rule, p. 309

$avg = \frac{1}{b-a}\int_a^b f(x)\,dx$

$f(x) = 2x$

$\int f(x)\,dx = x^2 + C$

$\int_1^2 f(x)\,dx = 3$

$F(2)+c - (F(1)+c)$

Chapter 6:

- Solving Differential Equations, p. 321, p. 350 (separation of variables)
- Slope Fields, p. 323
- Indefinite Integral
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- Newton's Law of Cooling, p. 355

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- Simpson's Rule, p. 309

endpts.

$$h = \frac{b-a}{n}$$

partitions

$$\frac{h}{2}\left(y_0 + 2y_1 + 2y_2 + \dots + 2y_{n-1} + y_n\right)$$

approximates integral $\int f(x)\,dx$

Chapter 6:

- Solving Differential Equations, p. 321, p. 350 (separation of variables)
- Slope Fields, p. 323
- Indefinite Integral
  - Properties of Indefinite Integrals, p. 332
  - U-substitution, p. 333
- Exponential Growth, p. 351
- ...f-life, p. 353

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- Simpson's Rule, p. 309

Chapter 6:

- Solving Differential Equations, p. 321, p. 350 (separation of variables)
- Slope Fields, p. 323
- Indefinite Integral
  - Properties of Indefinite Integrals, p. 332
  - U-substitution, p. 333
- Exponential Growth, p. 351
- Half-life, p. 353
- Newton's Law of Cooling, p. 355

$\frac{dy}{dx} = (xy)^2$ solve for y

if $y=1$ when $x=1$

$\frac{dy}{dx} = x^2y^2$

$\frac{dy}{y^2} = x^2dx$

$\int y^{-2}dy = \int x^2 dx$

$-\frac{1}{y} = \frac{x^3}{3} + C$

$y = -\frac{3}{x^3} + C$

Chapter 7:

- Integral as net change, p. 379
- Total Distance, p. 381
- Area between curves, p. 390
- Volume of solid, p. 399
  - Cross Sections, p. 400, 403
  - Washer Method, p. 401
  - Shell Method, p. 402

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$$y = Ae^{kt} \qquad \text{Let } A = \pm e^{C}.$$

This solution shows that the *only* growth function that results in a growth rate proportional to the amount present is, in fact, exponential. Note that the constant A is the amount present when $t = 0$, so it is usually denoted y_0.

The Law of Exponential Change

If y changes at a rate proportional to the amount present (that is, if $dy/dt = ky$), and if $y = y_0$ when $t = 0$, then

$$y = y_0 e^{kt}.$$

$2 = 1e^{k \cdot 12}$ $\quad \ln 2 = k \cdot 12$ $\quad k = \frac{\ln 2}{12}$

The constant k is the **growth constant** if $k > 0$ or the **decay constant** if $k < 0$.

$\frac{dy}{dx} = kt$

find k

doubles in 12yrs

Now try Exercise 11.

Continuously Compounded Interest

Suppose that A_0 dollars are invested at a fixed annual interest rate r (expressed as a decimal). If interest is added to the account k times a year, the amount of money present after t years is

$$A(t) = A_0\left(1 + \frac{r}{k}\right)^{kt}.$$

... be added ("compounded," bankers say) monthly ($k = 12$), weekly ($k =$ ... even more frequently, by the hour or by the minute.

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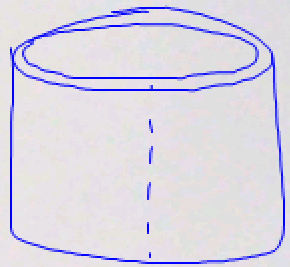
- Trapezoidal Rule, p. 307
- Simpson's Rule, p. 309

Chapter 6:

- Solving Differential Equations, p. 321, p. 350 (separation of variables)
- Slope Fields, p. 323
- Indefinite Integral
  - Properties of Indefinite Integrals, p. 332
  - U-substitution, p. 333
- Exponential Growth, p. 351
- Half-life, p. 353
- Newton's Law of Cooling, p. 355



Chapter 7:

- Integral as net change, p. 379
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  - Cross Sections, p. 400, 403
  - Washer Method, p. 401
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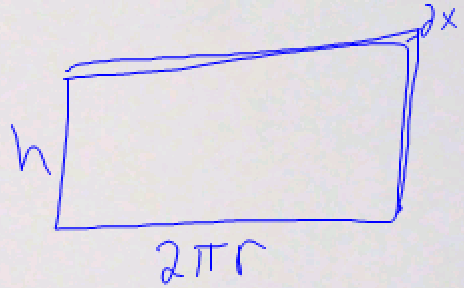


$$2\pi \int_a^b r \cdot h \, dx$$

This concludes the proof.

It is difficult to overestimate the power of the equation

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x). \qquad (1)$$

$$\frac{\partial}{\partial x}\int_3^{x^2}(2t+1)\,\partial t \rightarrow (2(x^2)+1)(2x)$$

It says that every continuous function f is the derivative of some other function, namely $\int_a^x f(t)\,dt$. It says that every continuous function has an antiderivative. And it says that the processes of integration and differentiation are inverses of one another. If any equation deserves to be called the Fundamental Theorem of Calculus, this equation is surely the one.

EXAMPLE 1 Applying the Fundamental Theorem

Find

$$\frac{d}{dx}\int_{-\pi}^{x}\cos t\,dt \quad \text{and} \quad \frac{d}{dx}\int_0^x \frac{1}{1+t^2}\,dt$$

by using the Fundamental Theorem.

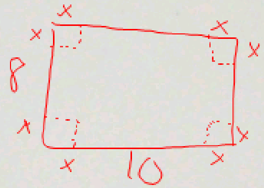
SOLUTION

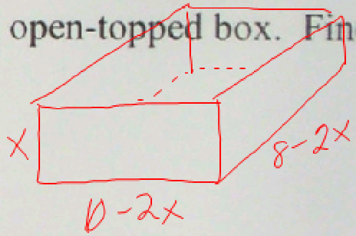
$$\frac{d}{dx}\int_{-\pi}^{x}\cos t\,dt = \cos x \qquad \text{Eq. 1 with } f(t) = \cos t$$

3) A rectangle is inscribed under the graph of $h(x) = 9 - x^2$ and above the x-axis. Find the maximum possible area for that rectangle.

4) An open-topped box with a square base must be constructed with a volume of 12 cubic inches. What dimensions use the least amount of material? (*Hint:* Define your variables for dimensions of the box. Now write two equations, one for volume and one for surface area. Which one will be optimized?)

5) From an 8 inch by 10 inch rectangular sheet of paper, square of equal size will be cut from each corner. The flaps will then be folded up to form an open-topped box. Find the maximum possible volume of the box.





$V = (x)(8-2x)(10-2x)$

$(x)(80-36x+4x^2)$

$48 - 144 + 80$

$V = 4x^3 - 36x^2 + 80x \longrightarrow V' = 12x^2 - 72x + 80$

$0 = 12x^2 - 72x + 80$

$0 = 4(3x^2 - 18x + 20)$

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$\approx 1.472, 4.528$

$V(1.472) \approx 52.514 \text{ in}^3$

$+ \quad - \quad +$

$1.472 \quad 4.528$

↓ max